Imaginary Powers of (k, a)-Generalized Harmonic Oscillator

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We will define and investigate the imaginary powers $(-\triangle_{k,a})^{-i\sigma}, \sigma \in \mathbb{R}$ of the (k, a)-generalized harmonic oscillator $-\triangle_{k,a} = -\|x\|^{2-a} \triangle_k + \|x\|^a$ for $a = 2$ and 1 repectively, and prove the L^p-boundedness $(1 < p < \infty)$ and weak L^1-boundedness of such operators. To prove this result, we develop the Calderón–Zygmund theory adapted to the (k, a)-generalized setting for $a = 2$ and 1, and show that $(-\triangle_{k,a})^{-i\sigma}$ are singular integral operators satisfying the corresponding Hörmander type condition.