Minimal polynomials with prescribed zeros

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Let E denote a compact subset of the complex plane $\mathbb{C}$ containing an infinite number of points. Then, there exists a unique monic polynomial of degree n that minimizes the infinity norm on E. This polynomial is known as the Chebyshev polynomial associated with E. In this presentation, we will explore a related problem by considering monic minimizers with respect to the infinity norm on the unit circle having prescribed zeros on the boundary. Building upon work in [3], we will see that prescribing a zero on the boundary dramatically changes the behavior of the corresponding minimizers in terms of the asymptotic zero distribution as well as the corresponding norms. Alternatively, this can be viewed as a weighted Chebyshev problem and we will extend the analysis to allow for fractional powers of the boundary zero in order to draw conclusions regarding Chebyshev polynomials corresponding to the lemniscatic family $\{z : |z^m - 1| = 1\}$. Finally, we will supplement our theoretical discussion with numerical experiments conducted using the complex Remez algorithm [5]. These experiments will serve to suggest directions for further study. This is based on joint work with A. Bergman, J. S. Christiansen and B. Eichinger.

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References

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