Scattering theory with both regular and singular perturbations and applications

Andrea Mantile

The mathematical scattering theory for short-range potential is a well studied subject which was developed by two essentially different approaches: the trace-class method and the smooth method. The scattering problem for singular perturbations of self-adjoint operators, which is outside the original scope of these methods, is connected with scattering from obstacles with impenetrable or semi-transparent boundary conditions. On this side, a general scheme has been developed by combining the construction of singular perturbations, following Posilicano's approach, with an abstract version of the Limiting Absorption Principle (LAP in the following) due to W. Renger and a variant of the smooth method due to M. Schechter. Let us recall that boundary triple theory and properties of the associated operator-valued Weyl functions were also used to obtain similar representation of the scattering matrix for singularly coupled self-adjoint extensions.

The target of this talk is to present a general framework for the multiple scattering with both potential type and singular perturbations. Our concern is the scattering theory with respect to the free Laplacian and the regular and the singular parts of the perturbation are dealt as a single object: this constitutes the main novelty of our approach. At first we provide an abstract resolvent formula for a perturbations A_{B} of the self-adjoint A by a linear combination of the adjoint of two bounded trace-like maps. The LAP for A_{B} and then an aymptotic completeness criterion for the scattering couple (A_{B}, A) are provided, under suitable hypothesis, as a generalisation of the scheme adopted for purely singular perturbations. Then, by a combination of LAP with stationary scattering theory in the Birman-Yafaev scheme and the invariance principle, we obtain a representation formula for the scattering matrix of the couple (A_{B}, A). Whenever A is the free Laplacian in $L^2(\mathbb{R}^3)$, such a formula contains, as subcases, both the usual formula for the perturbation given by a short-range potential and the formula for the case of a singular perturbation describing self-adjoint boundary conditions on a hypersurface.

After introducing the main features of this theory, we present applications where our construction is used to describe the stationary-scattering in composite acoustic or electromagnetic dispersive media. In particular, we show how the stationary resolvent and the scattering solutions, for scalar-wave equations with the divergence-form Laplacian having discontinuous density, can be represented in terms of a class of perturbations involving both potential terms and specific frequency-dependent interface conditions.