Multiple orthogonal polynomials in random matrix theory

Arno Kuijlaars

I will give an overview of some of the uses of multiple orthogonal polynomials (MOPs) in the theory of random matrices. Multiple orthogonal polynomials have orthogonality properties with respect to several orthogonality measures. They arise as averages of characteristic polynomials in a number of random matrix ensembles, including random matrices with external source, two matrix models, Muttalib-Borodin ensembles, and normal random matrices.

In such models, the limiting behavior of MOPs as their degrees tend to infinity is of interest for the eigenvalue behavior as the size of the random matrix increases. In typical examples, the limiting behavior of the zeros of the MOPs is given in terms of a vector equilibrium problem from logarithmic potential theory. New types of critical behavior and phase transitions appear beyond those that arise in models that are associated with ordinary orthogonal polynomials.