# Chebyshev polynomials and Widom factors

*Jacob S. Christiansen*

Let $\mathsf{E} \subset \mathbb{C}$ be an infinite compact set, and denote by $T_n$ the minimax (or Chebyshev) polynomials of $\mathsf{E}$, i.e., the monic degree $n$ polynomials minimizing the sup-norm on $\mathsf{E}$. A well-known result by Szegő asserts that $\|T_n\|_{\mathsf{E}} \geq \mathrm{Cap}(\mathsf{E})^n$ for all $n$, a lower bound that doubles when $\mathsf{E} \subset \mathbb{R}$, as proven by Schiefermayr. More recently, Totik proved that for real subsets, $\|T_n\|_{\mathsf{E}}/\,\mathrm{Cap}(\mathsf{E})^n \to 2$ if and only if $\mathsf{E}$ is an interval.

We will introduce the Widom factors, denoted by

$$W_n(\mathsf{E}) := \frac{\|T_n\|_{\mathsf{E}}}{\mathrm{Cap}(\mathsf{E})^n}$$

and investigate whether there exist additional subsets of $\mathbb{C}$ for which $W_n(\mathsf{E}) \to 2$. It appears that the answer is affirmative for certain polynomial preimages. Interestingly, our proof relies on properties of the Jacobi orthogonal polynomials established by Bernstein. We will also discuss the symmetry properties underlying this phenomenon and explore related open problems.

The talk is based on joint work with B. Eichinger (TU Wien) and O. Rubin (Lund).