Extremal polynomials on a Jordan curve or arc

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For a given Jordan curve or arc Γ in the complex plane a classic problem is to consider a measure μ supported on Γ, $z_0 \in \mathbb{C}$ and

$$\lambda_n(\mu, z_0) = \inf \left\{ \int |P|^2 d\mu \mid P \text{ is a polynomial of degree} \leq n \text{ and } P(z_0) = 1 \right\}.$$

This can be extended to $z_0 = \infty$ by considering the norm of the n-th monic orthogonal polynomial. Instead of L^2 it is also possible to consider the sup-norm on Γ. In both cases the right way to rescale is with the conformal mapping from the outside of Γ to the outside of the unit circle which is the notion of Widom factors. We will give a framework to characterize existing and new results which is related to the Szegő-function and Hardy space associated with a measure. We will also talk about the Ahlfors problem, a related polynomial minimization problem.