Ultrainvariant subspaces

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For an operator A on a complex Banach space X and a closed subspace $M \subseteq X$, the *local commutant* of A at M is the set $C(A;M)$ of all operators T such that $TAx = ATx$, for all vectors $x \in M$. It is clear that $C(A;M)$ is a closed space of operators, however, it is not an algebra, in general. One can show that $C(A;M)$ is an algebra if and only if the subspace $gir_A(M) = \{x \in X;\ TAx = ATx,\ \forall\ T \in C(A;M)\}$ is *ultrainvariant*, that is, invariant for every operator in $C(A;M)$. Every ultrainvariant subspace is hyperinvariant, but the opposite does not hold, in general. It is an open question of whether every operator with a non-trivial hyperinvariant subspace has a non-trivial ultrainvariant subspace.