Linear systems, differentials and determinants

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Let $(-A, B, C)$ be a continuous time linear system with state space a separable complex Hilbert space H, where $-A$ generates a strongly continuous contraction semigroup $(e^{-tA})_{t\geq 0}$ on H, and $\phi(x) = Ce^{-xA}B$ is the impulse response function. Let Γ_ϕ be the corresponding Hankel integral operator on $L^2(0,\infty)$. The paper introduces an algebra $\mathcal{E}$ of operators on H in which one solves the Lyapunov equation $dR_x/dx = -AR_x - R_xA$, so that $\det(I+R_0) = \det(I+\Gamma_\phi)$. The paper gives several determinant formulas related to the Carey–Pincus formulas for multiplicative commutators. Special results hold when the quotient $\mathcal{A}$ of $\mathcal{E}$ by the algebra of compact operators is quasi-free in the sense of Cuntz and Quillen [J. Amer. Math Soc. **8** (1995), 251-289]. Under suitable conditions on $(-A, B, C)$, this $\mathcal{A}$ gives a commutative and finitely generated algebra of differential operators such that the maximal ideal space of determines a hyperelliptic spectral curve. Work of Gordon Blower (School of Mathematical Sciences, Lancaster University, UK) and Ian Doust (UNSW Sydney, Australia)

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