On one class eigenvalue problems for fourth order differential operator equation

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We consider the next eigenvalue problem in space $L_2(H,(0,1))$

$$y^{IV}(t) + Ay(t) + q(t)y(t) = \lambda y(t)\,(1) \qquad (1)$$

$$y(0) = y"(0) = 0 \qquad (2)$$

$$-y'''(1) = \lambda Q_1 y(1) \qquad (3)$$

$$y"(1) = \lambda Q_2 y'(1) \qquad (4)$$

where H is abstract separable Hilbert space.Coefficients of equation are operators, namely A is unbounded self-adjoint operator with compact inverse , $A^{-1} \in \sigma_\infty$, and $q(t)$ is bounded in $H$for each $t \in [0,1]$ from which follows also boundedness of it in $L^2(0,1;H)$.Coefficients of boundary conditions $Q_1,\ Q_2$ are also unbounded selfadjoint , positive-definite operators in H. Obviously, one can't give operator formulation of that problem with some self-adjoint operator without leaving the space $L^2(0,1;H)$. Thus, our aim is definition of domain of minimal operator in exit space, give description of domains of maximal operator and self-adjoint extensions in terms of boundary conditions. And finally, give characterization of specrum, find asymptotics of spectrum and derive formula for regularized trace of operator in operator setting of boundary value problem (1)- (4).
Introduce the next direct sum of Hilbert spaces

$$\mathrm{H}{=}L_2(H,(0,1)) \bigoplus H_Q^2$$

Scalar product of its elements $Y = (y(t),\ y_1, y_2)\,,\ \ Z = (z(t),\ z_1, z_2)$ is defined by

$$(Y,Z)_{\mathrm{H}} = (y(t), z(t))_{L_2(H,(0,1))} + \left({Q_1}^{-1}y_1, z_1\right) + \left({Q_2}^{-1}y_2, z_2\right).$$

Let the operator L_0' has the domain

$$D(L_0') = \{Y \in \mathrm{H}\ /Y = (y(t),\ Q_1 y(1), Q_2 y'(1)),$$

$$y(t) \in C_0^\infty(H_\infty,(0,1])\,,\ y(1) \in D(Q_1)\,,\ y'(1) \in D(Q_2)\,,$$

$$y(1), y"(1), y"'(1)\ \in H\}\,,$$

where $H_\infty = \bigcap_{j=1}^\infty \left(D(A^j)\right)$, and

$$L_0'(Y) = (ly, -y'''(1), y"(1)\ ).$$

Closure of L_0' in H call the minimal operator . Its adjoint call maximal operator. We investigate spectral questions related to that opertors and their self-adjoint extensions.
Givent one relation between characteristic determinant and norming constants (reciprocal of norms of eigen-vectors) which is later applied to deriving the regularized trace formula.