Quantum Signal Processing and the nonlinear Fourier Transform

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Quantum Signal Processing (QSP) is an algorithmic process by which one represents a signal $f : [0,1] \to (-1,1)$ as the upper left entry of a product of $SU(2)$ matrices parametrized by the input variable $x \in [0,1]$ and some "phase factors" $\{\psi_k\}_{k\geq 0}$ depending on f. QSP was well-defined for polynomial signals f, but how to represent arbitrary signals $f : [0,1] \to (-1,1)$ was not fully understood till our recent work relating QSP to the nonlinear Fourier transform. We will see that, after a change of variables, QSP is actually the $SU(2)$ model of the nonlinear Fourier transform, and the phase factors $\{\psi_k\}_k$ correspond to the nonlinear Fourier coefficients. Then, by exploiting a nonlinear Plancherel identity and using some basic operator theory, we will show that QSP can be extended to all signals f satisfying the log integrability condition

$$\int_0^1 \log(1 - f(x)^2) \frac{dx}{\sqrt{1-x^2}} > -\infty \,.$$