Approximation with Neural Networks: Impact of Activation Functions

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In this talk, we explore the approximation capabilities of deep neural networks in two distinct contexts. First, we examine the challenges and solutions associated with different activation functions, focusing on the impact of using the Rectified Power Unit (RePU) activation for approximating Sobolev-regular functions.

Next, we delve into the practical challenges of computing these approximations from point samples, addressing the "theory-to-practice gap" in deep learning. Our study reveals that functions approximable by neural networks with RePU activation require an exponentially growing number of samples as the input dimension increases. This finding provides a comprehensive theoretical understanding of the sampling complexity bounds for neural network approximation spaces in connection with the chosen activation.

This presentation is based on a series of works including:

- "Sampling Complexity of Deep Approximation Spaces" by A. A. and Philipp Grohs, Analysis and Applications, (2023).
- "Approximations with Deep Neural Networks in Sobolev Time-Space" by A. A. and Philipp Grohs, Analysis and Applications, (2022).
- "Deep Neural Network Approximation for Hölder Functions" by A. A., arXiv:2201.03747, (2022).