Indefinite canonical systems and applications

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By de Branges inverse spectral theorem the set of all Nevanlinna functions (analytic functions q in the upper half plane with positive semidefinite kernel $N_q(w,z) := \frac{q(z)-\overline{q(w)}}{z-\overline{w}}$) corresponds via Weyl's limit point construction bijectively to the set of all positive semidefinite and trace-normalised Hamiltonians on the half line $(0,\infty)$.

M.G.Krein and H.Langer introduced and studied the class of generalised Nevanlinna functions (meromorphic function in the upper half plane whose kernel N_q has a finite number of negative squares). We discuss a generalisation of the notion of a canonical systems adapted to the setting of "finitely many negative squares", these are systems with a finite number of inner singularities behaving not too badly. Further we present some applications.

The task to find a notion of "indefinite canonical system" and prove a generalisation of de Branges' theorem was given to me by Heinz as the first thing when I started my PhD (I didn't understand a word then). It took about 15 years of hard work together with my friend and colleague M.Kaltenbäck to develop that theory.