Unitary boundary pairs for isometric operators in Pontryagin spaces, generalized coresolvents, and Kreĭn's formula

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Extension theory for symmetric and isometric operators in Pontryagin spaces was initiated by I.S. Iokhvidov and M.G. Kreĭn in the 1950s. Generalized resolvents of such operators were described by M.G. Kreĭn and H. Langer in early 1970s and later further developed by A. Dijksma, H. Langer, and H.S.V. de Snoo for (standard) isometric and symmetric relations even in the Kreĭn space setting. For a nonstandard isometric operator in a Pontryagin space, a description of its regular generalized resolvents in a Pontryagin space, without the growth of the negative index, was given by P. Sorjonen (1985), while the non-regular case allowing also a growth of the negative index was studied by O. Nitz (2000).

In the present talk we describe a coupling approach to study generalized coresolvents of isometric operators in the Pontryagin space setting. The methods used rely on a new general notion of boundary pairs for isometric operators in the Pontryagin space setting. Even in the Hilbert space case this notion generalizes the concept of boundary triples and associated Weyl functions for isometric operators introduced (more generally for dual pairs of operators) by M.M. Malamud and V.I. Mogilevskii (2003, 2004). The notion of boundary pairs for isometric operators offers an alternative approach to study operator valued generalized Schur functions without any additional invertibility requirements (at the origin). Combining this realization method for generalized Schur functions with the coupling method of boundary pairs, the generalized coresolvents of isometric operators via an analog of Krein'n formula can be obtained in the general case.

The coupling method was in fact initially appearing in order to give a simple solution to a problem introduced to me by Heinz Langer in Vienna 1992. That problem was concerning an invariance result of exit spaces when the exit spaces are constructed with the so-called ε-method simultaneously for a family of Nevanlinna functions, which were used as spectral parameters in the boundary conditions.

The talk is based on joint work with Vladimir Derkach and Dmytro Baidiuk.