

Positive and Negative Eigenfunction Expansion Results for Indefinite Sturm–Liouville Problems

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For a weight function $r \in L^1[-1, 1]$ with sign change at 0 we consider two indefinite regular Sturm–Liouville problems: $-f'' = \lambda r f$ with Neumann boundary conditions and $-\left(\frac{u'}{r}\right)' = \lambda u$ with Dirichlet boundary conditions. It is known that the eigenfunctions may (or may not) form a Riesz basis in the naturally associated weighted Hilbert spaces only simultaneously. Here, we study the situation where this property fails to hold. Nevertheless, we obtain eigenfunction expansions on smaller domains or in spaces with stronger or weaker topology. As the main result, we identify explicit functions in the “original” spaces which do not allow an eigenfunction expansion.