# Self-adjoint coupling, Straus subspaces, and the equation $\mathcal{W}(z)\mathcal{P}(z) \equiv \mathcal{P}(z)\mathsf{V}$

*Aad Dijksma*

The lecture concerns recent joint work with Branko Ćurgus (Bellingham).

In a coupling theorem from 2001 we described a special class of canonical self-adjoint extensions of the direct sum of symmetric linear relations $S_1$ and $S_2$ in Krein spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$ and assigned a unique parameter to each of these extensions. Assuming that $\mathfrak{H}_2$ is finite dimensional and that $S_2$ is an operator without eigenvalues, we construct a model for $(\mathfrak{H}_2, S_2)$ based on an essentially unique polynomial matrix $\mathcal{P}(z)$. The families of Straus subspaces associated with the self-adjoint extensions are characterized as restrictions of $S_1^*$ by polynomial boundary conditions involving $\mathcal{P}(z)$ and the parameters. We establish necessary and sufficient conditions on the parameters under which the extensions are similar and the corresponding families of Straus subspaces coincide. Related to our results is the equation $\mathcal{W}(z)\mathcal{P}(z) = \mathcal{P}(z)\mathsf{V}$ in which the unimodular matrix polynomial $\mathcal{W}(z)$ and the invertible matrix $\mathsf{V}$ are the unknowns. Explicit examples are given.