Hyper-positive functions and dissipativity

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Positive functions map the open right half-plane $\mathbb{C}_r$ into its closure and serve as a model for passive continuous-time, linear, time-invariant systems. Within this set, functions mapping $\mathbb{C}_r$ into a sub-region of $\mathbb{C}_r$, which can be contained in a finite disk, can be identified with quantitative dissipativity. Such functions are called hyper-positive and, in the scalar case, are characterized by an inequality

$$F(z) + F(z)^* \geq \beta(1 + |F(z)|^2), \quad z \in \mathbb{C}_r$$

for some $\beta \in (0, 1)$. In the talk we consider such functions in the matrix-valued setting, replacing the above condition by the positive-definiteness of the kernel

$$\frac{F(z) + F(w)^* - T - F(z)TF(w)^*}{z + \overline{w}}.$$

where $T \in \mathbb{C}^{n \times n}$ satisfies $0 < T < I_n$. We present their main properties such as minimal realizations in the rational case, and study the associated reproducing kernels spaces. Connections to stability in the spirit of the Kalman-Yakubovich-Popov lemma are also presented.

This is joint work with Izchak Lewkowicz (Ben-Gurion University of the Negev, Beer-Sheva, Israel).

References:

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